Exercise 36

Invasive species often display a wave of advance as they colonize new areas. Mathematical models based on random dispersal and reproduction have demonstrated that the speed with which such waves move is given by the function $f(r) = 2\sqrt{Dr}$, where r is the reproductive rate of individuals and D is a parameter quantifying dispersal. Calculate the derivative of the wave speed with respect to the reproductive rate r and explain its meaning.

Solution

Take the derivative of f with respect to r by using the chain rule.

$$\frac{df}{dr} = \frac{d}{dr}\left(2\sqrt{Dr}\right)$$

$$= 2\frac{d}{dr}\sqrt{Dr}$$

$$= 2 \cdot \frac{1}{2}(Dr)^{-1/2} \cdot \frac{d}{dr}(Dr)$$

$$= (Dr)^{-1/2} \cdot (D)$$

$$= \frac{1}{\sqrt{Dr}} \cdot D$$

$$= \sqrt{\frac{D}{r}}$$

This represents the rate that the wave speed changes as the reproductive rate increases.